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SESSION I: INERTIAL SYSTEMS AND SENSORS

DEMODULATED DYNAMICS AND OPTIMAL FILTERING FOR CORIOLIS VIBRATORY GYROSCOPES *

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Abstract

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In this paper, a novel approach to CVG dynamics analysis in demodulated signals is proposed. Generalised differential equations of different types of CVG in amplitude-phase variables are then used to produce system transfer functions that relate unknown angular rate, which becomes now an input to the system, to the amplitude and phase of secondary oscillations. Using these transfer functions, different optimal filters are derived using Wiener-Kolmogorov approach both for output added noises and stochastic disturbances. Efficiency of these filters is studied for different noise-to-signal ratios. Accuracy of derived transfer functions as well as efficiency of obtained optimal filters for CVG is demonstrated by means of numerical simulation using Simulink/Matlab, where filters are applied to realistically modelled gyroscope after synchronous demodulation of its output.

Introduction

Significant amount of interest received by Coriolis vibratory gyroscopes (CVGs) from both the scientific and engineering communities is due to the possibility to fabricate sensitive elements of such gyroscopes in miniature form by using modern microelectronic mass-production technologies. Being based on sensing of Coriolis acceleration due to the rotation in oscillating structures, CVGs have a lot more complicated mathematical models, comparing to the conventional types of gyroscopes. One of such complication is a result of the useful signal proportional to the external angular rate being modulated with the intentionally excited primary oscillations [1-3]. From the control systems point of view, conventional representation of CVGs incorporates primary oscillation excitation signal as an input to the dynamic system, and unknown angular rate as a coefficients of its transfer functions [3]. As a result, conventional control and filtering systems design is practically impossible. At the same time, performances of CVGs are limited mainly due to the low signal-to-noise ratios. In view of this problem, optimal noise filter development is highly necessary. The latter could be achieved only in systems where unknown angular rate is no longer a system parameter but its input.

Demodulated dynamics of Coriolis vibratory gyroscopes

In the most generalized form, motion equations of the CVG sensitive element both with translational and rotational motion could be represented in the following form [4]:

$$\begin{cases} \ddot{x}_1 + 2\zeta_1 k_1 \dot{x}_1 + (k_1^2 - d_1\Omega^2)x_1 + g_1\Omega\dot{x}_2 + d_3\dot{\Omega}x_2 = q_1(t), \\ \ddot{x}_2 + 2\zeta_2 k_2 \dot{x}_2 + (k_2^2 - d_2\Omega^2)x_2 - g_2\Omega\dot{x}_1 - \dot{\Omega}x_1 = q_2(t). \end{cases} \quad (1)$$

Here x_1 and x_2 are the generalized coordinates that describe primary (excited) and secondary (sensed) motions of the sensitive element respectively, k_1 and k_2 are the corresponding natural frequencies, ζ_1 and ζ_2 are the dimensionless relative damping coefficients, Ω is the measured angular rate, which is orthogonal to the axes of primary and secondary motions, q_1 and q_2 are the generalized accelerations due to the external forces acting on the sensitive element. The remaining dimensionless coefficients are different for the sensitive elements exploiting either translational or rotational motion. For more details on the calculating coefficients of the system (1) see [4].

In order to make the equations (1) suitable for to the transfer function synthesis one must make the following assumptions: angular rate is small comparing to the primary and secondary natural frequencies, and rotational and Coriolis accelerations acting along primary oscillation axis are negligible in comparison to the accelerations from driving forces. Using the amplitude-phase substitutions for primary and secondary generalized displacements of CVG sensitive element, the following motion equations expressed in terms of complex amplitudes can be obtained:

$$\begin{cases} \ddot{A}_1 + 2(\zeta_1 k_1 + j\omega)\dot{A}_1 + (k_1^2 - \omega^2 + 2j\omega k_1\zeta_1)A_1 = q_{10}, \\ \ddot{A}_2 + 2(\zeta_2 k_2 + j\omega)\dot{A}_2 + (k_2^2 - \omega^2 + 2j\omega k_2\zeta_2)A_2 = (j\omega g_2\Omega + \dot{\Omega})A_1 + g_2\dot{A}_1\Omega. \end{cases} \quad (2)$$

Equations (2) describe variations of the amplitude and phase of the primary and secondary oscillations in time with respect to the unknown non-constant angular rate $\Omega(t)$. This allows conducting analysis of the Coriolis vibratory gyroscope dynamics without constraining the angular rate to be constant or slowly varying.

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System transfer functions

Having CVG sensitive element motion equation in the form (2) allows obtaining its transfer functions from the input angular rate to the amplitude of the secondary oscillations.

$$W_2(s)=\frac{A_2(s)}{\Omega(s)}=\frac{A_1(s+jg_2\omega)}{(s+j\omega)^2+2\zeta_2k_2(s+j\omega)+k_2^2}. \tag{3}$$

One should note that transfer function (3) has complex coefficients, which results in the complex system outputs as well. There is quite an important special case, when complex transfer function (3) transform to the simple real-valued one. Assuming equal primary and secondary natural frequencies ($k_1=k_2=k$), equal damping ratios ($\zeta_1=\zeta_2=\zeta$), resonance excitation ($\omega=k$), and constant angular rate, one can easily obtain

$$W_{20}(s)=\frac{A_{20}(s)}{\Omega(s)}=\frac{q_{10}g_2}{4k^2\zeta(s+k\zeta)}. \tag{4}$$

Transfer function (4) relates angular rate to the secondary oscillations amplitude. However, more appropriate would be to consider transfer function relating unknown input angular rate to the measured angular rate, which can be easily obtained from (4) by dividing it on the steady state scale factor. The resulting transfer function is

$$W_\Omega(s)=\frac{k\zeta}{s+k\zeta}. \tag{5}$$

Although this case appears to be very specific, it still approximates transient process of a "tuned" CVG with accuracy suitable for most of applications [6, 7].

Optimal filter synthesis

Performances of CVGs can be affected by uncontrolled stochastic influences in two ways: as a "sensor noise", which is added to the output of the system, and as a "process noise" or disturbances, which are added to the input of the system. The latter could be also treated as "rate-like" disturbances. Assuming that CVG is installed on a moveable object, such as aircraft or land vehicle, its angular rate power spectral density can be represented as

$$S_\Omega(s)=\frac{\sigma^2B^2}{B^2-s^2}, \tag{6}$$

where B is the moveable object bandwidth. In this case disturbances can be represented either as a white noise, or the high pass noise, adjacent to the object bandwidth. Assuming the minimal error variances integral performance criterion, and finding the structure and parameters of the filter transfer functions, that minimises this performance functional, the following optimal filters for the stochastic disturbances were obtained:

$$G(s)=\frac{B\sqrt{1+\gamma^2}(s+\zeta k)}{\zeta k(\gamma s+B\sqrt{1+\gamma^2})}, \tag{7}$$

in case of the "white-noise" disturbances and

$$G(s)=\frac{B(s+\zeta k)}{\zeta k(B+\gamma s)}, \tag{8}$$

in case of the "high-pass" disturbances. Here γ is the noise-to-signal ratio. Depending on which of the disturbance model is found to be the most appropriate, either filter (7) or filter (8) should be used.

Let us now consider the case of a "sensor" noise, when the noise is added to the output of the gyroscope. In case of the "white" noise added to the CVG output, optimal filter is given by the following expression:

$$G(s)=\frac{B\sqrt{1+\gamma^2}(s+\zeta k)}{\gamma s^2+s\sqrt{\gamma(B^2\gamma+\zeta^2k^2\gamma+2\zeta kB\sqrt{1+\gamma^2})}+\zeta kB\sqrt{1+\gamma^2}}. \tag{9}$$

Performance of the latter filter (9) is shown in the figure 1 below.

In these simulations optimal filter (9) is applied to the realistically modelled gyroscope output with added white noise. To build numerical model of CVG, motion equations (1) were used.

Conclusions

The presented above synthesis of the stochastic disturbances filters resulted in static filters capable of improving the performances of Coriois vibratory gyroscopes in the presence of "white" and "high-pass" process noise, as well as the sensor noise.

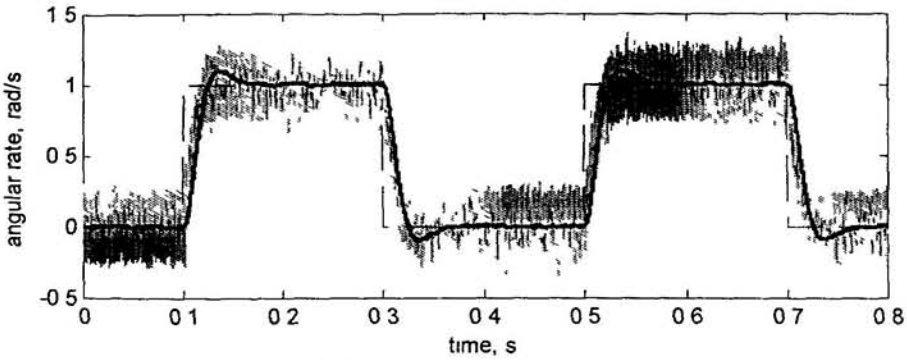

Fig. 1. Sensor noise filtering simulations
(dashed – input angular rate, gray – noised output, black – filtered output)

The latter has been demonstrated using explicit numerical simulations. The further analysis of the sensitivity of the filters performances in case of varying parameters of gyroscopes is viewed as a possible future development of the current research.

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