Whole Angle Force Rebalance Control for Coriolis Vibratory Gyroscopes

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***Abstract*—Synthesis of a feedback control system for Coriolis vibratory gyroscopes (CVG) that implements whole angle force rebalance mode of operation is presented in this paper. Feedback controller is derived in terms of demodulated signals. Successful operation of the control system is demonstrated using numerical simulation of realistic CVG model in modulated signals.**

Keywords—Coriolis; gyroscope; whole angle; force rebalance; feedback control

I. Introduction

Coriolis vibratory gyroscopes (CVG) are found in many applications nowadays, especially when they are implemented in miniature forms using micro-machining technologies [1, 2]. CVGs are used as an angular rate sensors both in control and navigation systems. Needless to say that requirements to CVG performances vary among different applications. The necessity to measure angle of rotation instead of angular rate (whole angle measurement mode) led to specifically designed sensitive elements, behaving similarly to Foucault pendulum [3]. In this paper different approach is considered, when a conventional angular rate sensing CVG is provided with a feedback controller, providing similar whole angle operation while restraining rate sensing output of the sensitive element.

II. Dynamics Of The Sensitive Element

Generalised motion equations of the CVG sensitive element are as follows:

$$\begin{cases} \ddot{x}_1 + 2\zeta_1 k_1 \dot{x}_1 + (k_1^2 - d_1\Omega^2)x_1 = q_1 - g_1\Omega\dot{x}_2 - d_3\dot{\Omega}x_2, \\ \ddot{x}_2 + 2\zeta_2 k_2 \dot{x}_2 + (k_2^2 - d_2\Omega^2)x_2 = q_2 + g_2\Omega\dot{x}_1 + d_4\dot{\Omega}x_1. \end{cases} \quad (1)$$

Here x_1 and x_2 are the generalised displacements describing primary and secondary motion of the sensitive element, either translational or rotational; k_1 and k_2 are the natural frequencies; ζ_1 and ζ_2 are the damping factors of the primary and secondary motions correspondingly; q_1 and q_2 are the accelerations (either translational or rotational) from external forces/torques; Ω is the external angular rate, orthogonal to the primary and secondary motions. Remaining coefficients are the functions of the sensitive element design parameters and could be found in [4].

Using the following complex amplitude-phase variables:

$$A_i(t) = A_{i0}(t)e^{j\varphi_{i0}(t)},$$

where i equals 1 or 2 for the primary or secondary oscillations correspondingly, A_{i0} and φ_{i0} are the amplitudes and phases of the primary and secondary oscillations, and substituting them into the motion equations (1), in case of the slowly varying secondary amplitude ($\ddot{A}_2 \approx 0$), second equation in (1) can be transformed to the following form

$$\begin{aligned} &2(\zeta_2 k_2 + j\omega)\dot{A}_2 + (k_2^2 - \omega^2 + 2j\omega k_2\zeta_2)A_2 \\ &= (j\omega g_2\Omega + d_4\dot{\Omega})A_1. \end{aligned} \quad (2)$$

Here ω is the excitation frequency of the primary oscillations, and

$$A_1 = \frac{q_{10}}{k_1^2 - \omega^2 + 2jk_1\zeta_1\omega}$$

is the constant in time complex amplitude of the primary oscillations, where q_{10} is the amplitude of the acceleration from the harmonic excitation forces. Influence of the secondary oscillations on the primary has been neglected in comparison to the excitation forces. In case of perfectly matched natural frequencies ($k_1 = k_2 = k$) and damping ($\zeta_1 = \zeta_2 = \zeta$), measured by CVG angular rate can be represented in Laplace domain as

$$\Omega^*(s) = W_\Omega(s)\cdot\Omega(s), \quad (3)$$

where

$$W_\Omega(s) = \frac{k\zeta}{s + k\zeta} \quad (4)$$

and describes dynamics of a CVG in demodulated (envelope) signals with respect to the unknown angular rate, which is now an input [5].

III. Feedback Controller Synthesis

In terms of the demodulated (envelope) signals, CVG along with the negative feedback loop can be represented as a control system shown in Fig. 1.

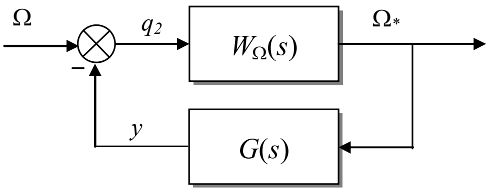

Fig. 1. CVG with the feedback controller

Here Ω_* - is the actual output of the gyroscope (measured angular rate), Ω - is the unknown angular rate (system input). The goal is to design such a feedback controller $G(s)$, producing output y, which is being modulated and applied to the secondary mode of the CVG sensitive element will limit actual output of the gyro. At the same time, signal y itself must represent integrated angular rate, e.g. angle of rotation.

Transfer function from the input angular rate Ω to the feedback output y is as follows:

$$W_y(s) = \frac{W_\Omega(s)\cdot G(s)}{1+W_\Omega(s)\cdot G(s)}. \quad (5)$$

In order to perform the required task of the integrated angular rate measurement ("whole angle" mode), transfer function (5) must be equal to the simple integrator $1/s$ and resulting in the following equation:

$$\frac{W_\Omega(s)\cdot G(s)}{1+W_\Omega(s)\cdot G(s)} = \frac{1}{s}. \quad (6)$$

Substituting (4) into the equation (6) and solving this equation for the unknown feedback transfer function $G(s)$ results in

$$G(s) = \frac{\zeta k + s}{\zeta k(s-1)}. \quad (7)$$

Note, transfer function (7) derived in terms of demodulated envelope signals. It means, that in order to apply its output as an actuation to the secondary mode of the sensitive element (acceleration q_2 in (1)), it must be modulated with the differentiated output of the primary mode as follows, to make it identical to the Coriolis force, acting along the secondary motion coordinate:

$$q_2(t) = g_2\cdot y(t)\cdot \dot{x}_1(t). \quad (8)$$

Applying signal (8) to the sensitive element results in reduction of its displacements, while the feedback output y becomes new output of the gyro, implementing whole angle operation.

IV. Results of Simulations

Simulation schematics of CVG with the feedback controller is shown in Fig. 2. Here subsystem "CVG dynamics" simulates sensitive element dynamics based on the equations (1). Demodulated rate like outputs of the CVG are shown in Fig. 3. One could note that the actual output of the gyro (solid line) is actually less than the input angular rate. At the same time, output of the feedback controller (shown in Fig. 4) produces integrated angular rate (angle of rotation) while reducing effect from the noise in the feedback loop.

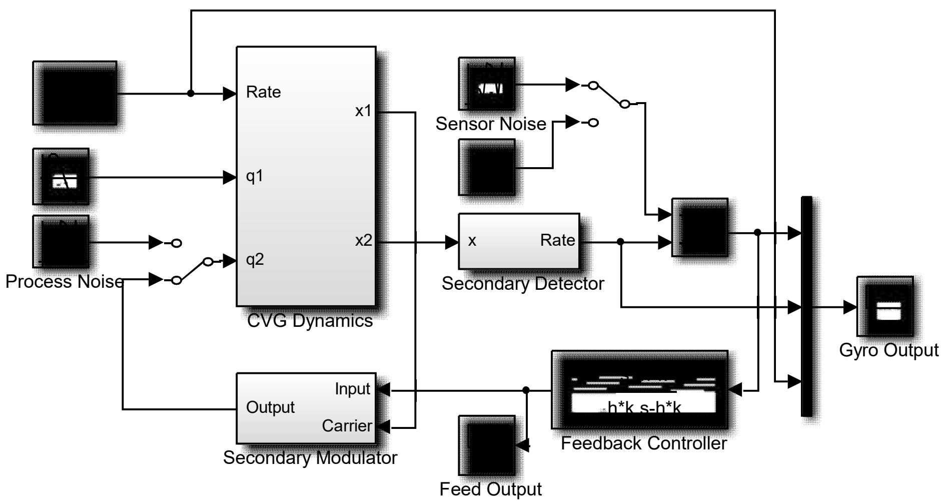

Fig. 2. Simulating CVG control operation in Simulink

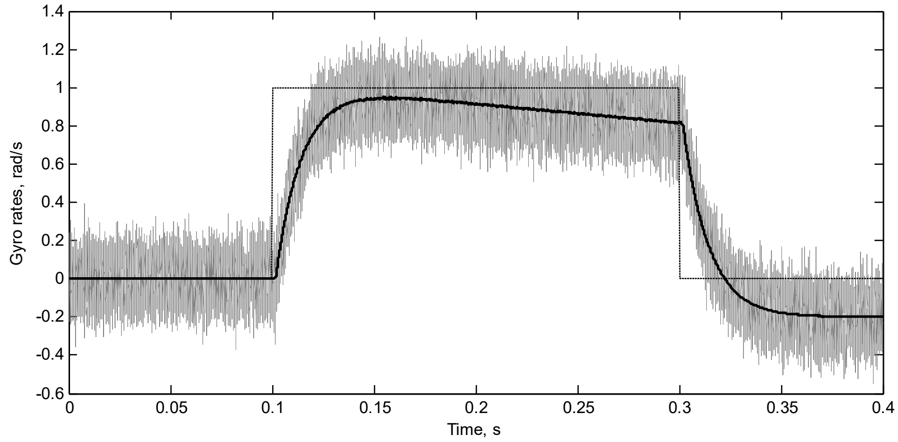

Fig. 3. CVG signals: solid - secondary output in the force rebalance mode, gray - noised secondary output, dashed - input angular rate

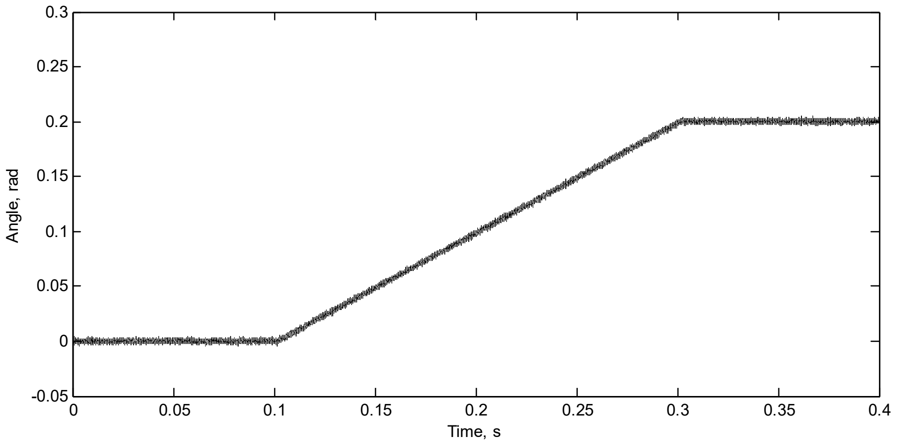

Fig. 4. Integrated (whole angle) CVG feedback output

V. Conclusions

Presented synthesis of a feedback controller results in a system, which being applied to a conventional CVG allows to implement whole angle force rebalance mode for the gyro. Obtained controller reduces sensitive element deflections and reduces influence of the measurement noise on the output angle of rotation as well. Optimal synthesis of the feedback controller in presence both of the process and sensor noises is considered as topic for the future research.

References

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