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Whole Angle Approximations of Aerodynamic Coefficients

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***Abstract*—This paper addresses the whole angle approximations of aerodynamic coefficients. Several nonlinear approximations are proposed to describe lift, drag and side force coefficients in the range from 0 to 360 degrees based on accuracy and simplicity.**

Keywords*—*flight simulations; aerodynamic coefficients; nonlinear approximations

I. INTRODUCTION

New improvements in modern aircraft designs resulted in the need for highly sophisticated mathematical models of aircraft dynamics. Such a model could not be developed without corresponding lift, drag and side force representations that will be both accurate and relatively simple. At the same time, simple and accurate models of aircraft dynamics are highly necessary for flight control and navigation system research and development, and for flight simulations in both civil aviation and military trainings. The aerodynamic model is possibly the most critical element of a flight simulator. Although conventional and currently widely used linear approximations of lift and drag coefficients deliver good results for the small incidence angle, they expectedly fail long before the incidence angle even reaches its stall value [1].

In [1] several harmonic approximations of lift and drag coefficients are compared and analyzed. The best representation in terms of accuracy and simplicity is found and proposed, and the problem of its parameters estimation is solved. In this paper are compared several nonlinear approximations of side force coefficients.

II. AERODYNAMIC FORCES AND COEFFICIENTS

Let us consider two reference systems with the same origin O at the center of gravity. Fixed-body system $X_b Y_b Z_b$ has the X_b axis pointing forward out of the nose of the aircraft, the Y_b axis pointing out the starboard wing and the Z_b axis pointing down. And velocity reference system $X_w Y_w Z_w$, where $\vec{V}$ - the total velocity vector, as shown in Fig.1. The aerodynamic forces depend only upon the angles α (incidence angle) and β (sideslip angle), which orient the total velocity vector $\vec{V}$ in relation to the axis X_b .

Viewed from above (in plan form), most aircrafts are symmetric. However, if the fuselage is not aligned with the direction of flight, it will make an incident angle with the wind, known as the angle of sideslip, β. Although the side force generated will be considerably less than the lift force on an aircraft, it is still significant.

Under these conditions, the fuselage acts as a large airfoil, with an angle of incidence β to the wind. Consequently, lift is generated by the fuselage but in the direction along the wing, causing a lateral force, known as side force.

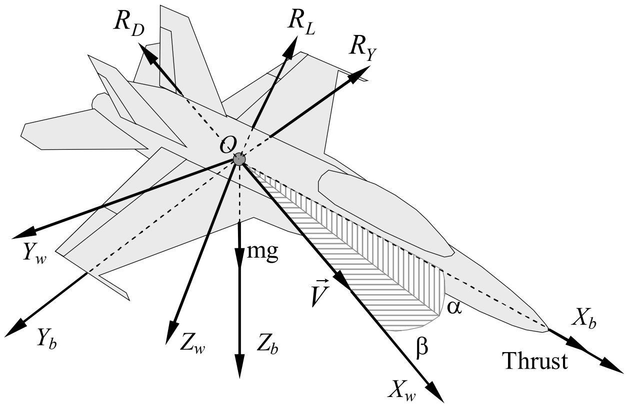


Figure 1. Aerodynamic forces

For the essentially subsonic velocities, aerodynamic lift, drag and side forces are usually calculated as [2]:

$$R_L = \frac{\rho V^2 S}{2} C_L(\alpha), R_D = \frac{\rho V^2 S}{2} C_D(\alpha),$$
$$R_Y = \frac{\rho V^2 S}{2} C_Y(\beta) \tag{1}$$

where $C_Y(\beta)$ is side force coefficient, $C_L(\alpha)$ is the lift coefficient, $C_D(\alpha)$ if the drag coefficient, $C_Y(\beta)$ is the side force coefficient, ρ is the air density at the given flight altitude, and S is the characteristic wing area.

Using the conventional approach and assuming small incidence angles, the lift coefficient is usually linearly approximated as:

$$C_L(\alpha) = C_{L_0} + C_{L_\alpha}\alpha \tag{2}$$

where C_{L_0} is the value of C_L for zero incidence angle, $C_{L_\alpha} = \partial C_L / \partial\alpha$ is the change in airplane lift force coefficient due to a change in incidence angle.

For the same small incidence angles, the aerodynamic drag coefficient is represented in the following form:

$$C_D(\alpha) = C_{D_0} + C_{D_\alpha}\alpha \tag{3}$$

where C_{D_0} is the value of C_D for zero incidence angle, $C_{D_\alpha} = \partial C_D / \partial\alpha$ is the change in airplane drag force coefficient due to a change in incidence angle.

The side force coefficient is usually linearly approximated as:

$$C_Y(\beta) = C_{Y_0} + C_{Y_\beta}\beta \tag{4}$$

where C_{Y_0} is the value of C_Y for: $\alpha = \beta = 0$, $C_{Y_\beta} = \partial C_Y / \partial\beta$ is the change in airplane side force coefficient due to a change in angle of sideslip (at constant angle of attack) [2].

In reality the aerodynamic drag coefficient is dependent not only on the incidence angle, but also on sideslip angle. Similarly to the longitudinal motion harmonic approximation of the drag coefficient in [1], in case of the both incidence and side-slip angles present the following approximation can be suggested:

$$\begin{aligned} C_D(\alpha,\beta) &= d_0 + d_1\cos(2\alpha) + d_2\cos(4\alpha) + \\ &+ d_3\cos(2\beta) + d_4\cos(4\beta) \end{aligned} \tag{5}$$

Coefficients d_0, d_1, d_2, d_3, d_4 could be easily calculated using the same approach as for the side force coefficient, for this purpose it could be used wind tunnel data or data obtained using CFD tools.

Because all of the preceding approximations (2-4) are true for the small incidence angles, it is not suitable for the whole incidence angle (360-deg) modeling. In this case, other approximations need to be proposed and investigated, that was done in [1] for the lift and drag coefficients and in this research nonlinear approximations was proposed for the side force coefficient.

III. Different Nonlinear Approximations

Considering the fact that all of the aerodynamic coefficients are naturally 2π periodic, approximations using sine and/or cosine functions are the first and obvious choice [1].

For the investigation of different approximation approaches for side force coefficient data from [3] is used. How side force coefficient depends on angle of attack and sideslip angle is shown in Fig. 2.

As one can from the Fig.2 surface of the side force coefficient under different α and β has complex shape. So, the approximations for this coefficient investigated in this paper are as follows:

$$C_Y = \sin((k_1\beta)(s_1\sin(k_2\beta + c_2) + s_2(\cos(\alpha + c_1)))) \tag{6}$$

$$C_Y = \sin((k_1\beta)(s_2(\sin(\alpha + c_1))^2)) \tag{7}$$

$$C_Y = \sin((k_1\beta)(s_1\sin(k_2\beta + c_2) + s_2(\sin(\alpha + c_1))^5)) \tag{8}$$

$$C_Y = \sin((k_1\beta)(s_2(\sin(\alpha + c_1))^3)) \tag{9}$$

where α is the incidence angle in radians, β is sideslip angle in radians, k_1, k_2, s_1, s_2, c_1, c_2 are independent parameters for the side force coefficient, that are to be determined based on the best of experimental data.

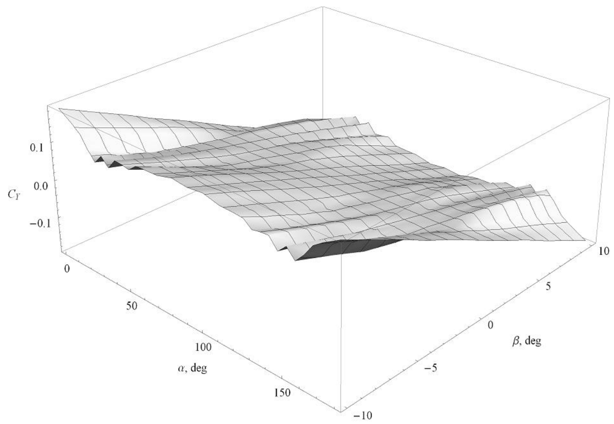


Figure 2. Side force under different α and β

IV. Approximation Parameters Calculation

The set of approximation functions was considered during the research.

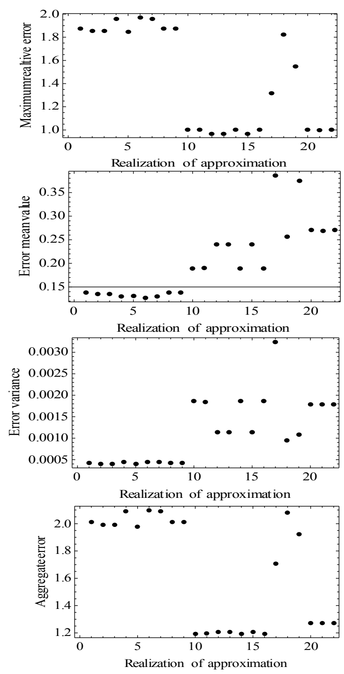


Figure 3. Characteristics of errors for different approximations

Figure 4. Characteristics of errors for different approximations

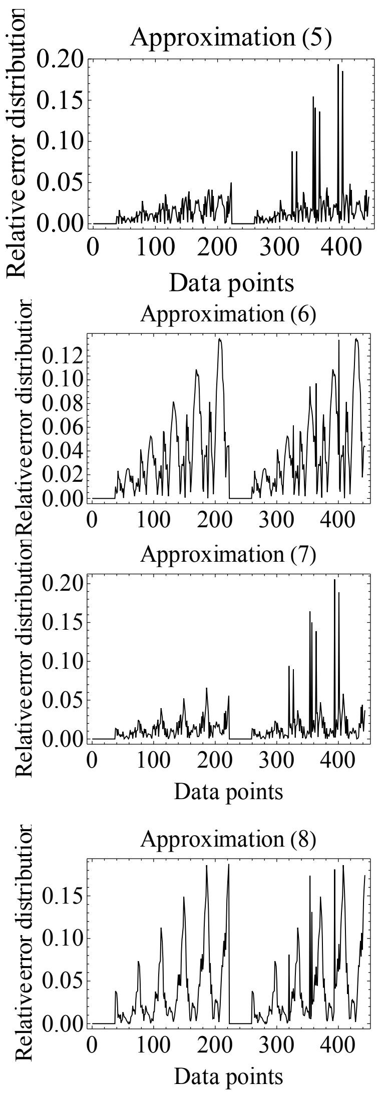
For each function was calculated parameters, which was best fitted into the experimental data using the nonlinear regression method. For every function it was evaluated mean relative error, maximum relative error, variance and it's aggregate characteristic according to the experimental data, which were used from [3].

Let's define non-dimensional characteristics presented in Fig. 3.

Maximum relative error is defined as follows:

$$MRE = \frac{\max \sqrt{(C_{Ya} - C_{Ye})^2}}{C_{Ym}}$$

where C_{Ya} is vector of approximated values, C_{Ye} is vector of experimental data and C_{Ym} is value of C_{Ye} with highest absolute error.



Error mean value is defined by the following formula:

$$RME = \frac{mean\sqrt{(C_{Ya} - C_{Ye})^2}}{C_{Ym}}$$

Variance is defined in the following way:

$$RVE = mean(C_{Ya} - C_{Ye})^2$$

For this research aggregate error was defined as:

$$A = MRE + RME + RVE$$

Functions (6-9) have distribution of errors for the given experimental data as shown in the Fig. 4; relative error distribution is deviation of the approximated values from the experimental values normalized by the experimental values.

V. CONCLUSIONS

Empirical aerodynamic side force coefficient representations (6-9) allow expanding of existing linear representation (4) to the whole range of possible incidence and sideslip angles. Such improved harmonic representations could easily replace the conventional linear representations and deliver better results. Also, its necessary to mention that these approximations was made only for the one set of experimental data and as future research obtained formulas should be checked on the different sets of experimental data or, as alternative, on the data obtained from CFD tools.

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