# Modelling Temperature Errors in Coriolis Vibratory Gyroscopes

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*Abstract*— **Effect of temperature variations on the output of Coriolis vibratory gyroscope is studied in this paper. Mathematical model of the temperature influences is developed and its parameters are identify using experimental data. The latter has been used to validate obtained model**

*Index Terms*—**Coriolis vibratory gyroscope, cross-damping, modeling, temperature errors.**

## I. INTRODUCTION

Vibratory gyroscopes utilizing Coriolis effect were successfully used in vast amount of different applications since micro fabrication techniques made possible to reduce its cost in mass production along with significant reduction in size [1, 2].

At the same time, Coriolis vibratory gyroscopes (CVG) traditionally occupy niche of low accuracy sensors due to the low stability of its performances under influence of operational environment factors. One of the major sources of such instabilities is temperature variations that cause changes in all measurement characteristics of CVGs [3, 4]. In this paper we study the effect of temperature variations on CVG, develop empirical model of the temperature influences, identify its parameters, and develop model of angular rate measurement error due to the temperature variations. Later we validate obtained models using experimental data obtained for CVG with cylindrical sensitive element.

## II. TEMPERATURE RELATED ZERO-RATE OUTPUT

Significant temperature related zero-rate output has been observed during experimental tests of CVG. For the temperature profile, shown in Fig. 1, and zero angular rate, CVG output is shown in Fig. 2 (uncompensated).

It is assumed that temperature variations cause this bias through the temperature dependent cross-damping. In this case excited primary oscillations of the sensitive element will induce secondary (output) oscillations even without external rotation being applied to the sensor. Moreover, cross-damping induced oscillations will not be distinguishable from the oscillations due to the angular rate.

In order to develop mathematical model for this phenomenon let us first analyze how cross-damping affects dynamics of the CVS sensitive element.
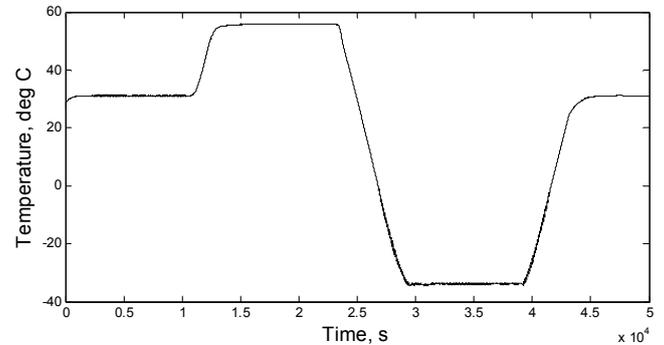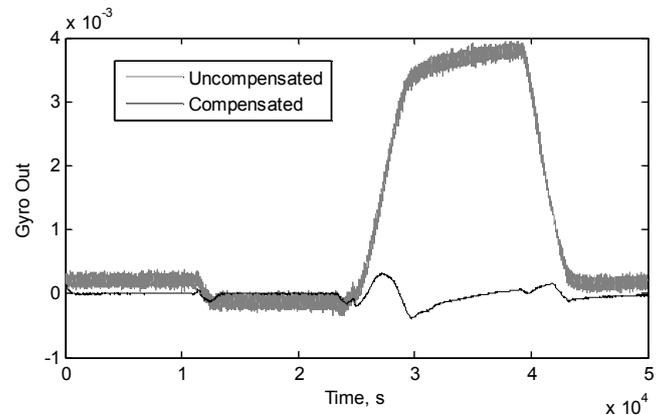


Fig. 1. Temperature profile



Fig. 2. CVG output with and without temperature compensation

### Fig. 3. SENSITIVE ELEMENT MOTION EQUATIONS

In the most generalized form, motion equations of the CVG sensitive element both with translational and rotational motion could be represented in the following form [5]:

$$\begin{cases} \ddot{x}_1 + 2\zeta_1 k_1 \dot{x}_1 + (k_1^2 - d_1\Omega^2)x_1 + g_1\Omega\dot{x}_2 + d_3\dot{\Omega}x_2 = q_1(t), \\ \ddot{x}_2 + 2\zeta_2 k_2 \dot{x}_2 + (k_2^2 - d_2\Omega^2)x_2 - g_2\Omega\dot{x}_1 - \dot{\Omega}x_1 = q_2(t). \end{cases} \quad (1)$$

Here $x_1$ and $x_2$ are the generalized coordinates that describe primary (excited) and secondary (sensed) motions of the sensitive element respectively, $k_1$ and $k_2$ are the corresponding natural frequencies, $\zeta_1$ and $\zeta_2$ are the

dimensionless relative damping coefficients, $\Omega$ is the measured angular rate, which is orthogonal to the axes of primary and secondary motions, $q_1$ and $q_2$ are the generalized accelerations due to the external forces acting on the sensitive element. The remaining dimensionless coefficients are different for the sensitive elements exploiting either translational or rotational motion. For the vibrating cylinder sensitive element, for example, $d_1 = d_2 = 1$, $d_3 = 1$, $g_1 = 2$, $g_2 = 2$. For other sensitive elements designs expressions for these coefficients can be found in [5].

If cross damping is present in the system, the motion equations 1 are transformed to the following form

$$\begin{cases} \ddot{x}_1 + 2\zeta_1 k_1 \dot{x}_1 + (k_1^2 - d_1\Omega^2)x_1 = \\ = -(g_1\Omega + 2\zeta_{21}k_1)\dot{x}_2 - d_3\dot{\Omega}x_2 + q_1(t), \\ \ddot{x}_2 + 2\zeta_2 k_2 \dot{x}_2 + (k_2^2 - d_2\Omega^2)x_2 = \\ = (g_2\Omega + 2\zeta_{12}k_2)\dot{x}_1 + \dot{\Omega}x_1 + q_2(t). \end{cases} \quad (2)$$

Here $\zeta_{12}$ and $\zeta_{21}$ are the relative cross-damping coefficients. Constant cross-coupling through the damping can be removed by calibration. However, calibration is unable to deal effectively with the varying in time damping.

### III. CROSS-DAMPING INDUCED OUTPUT

Let us first analyze cross-damping related components in the amplitude of the secondary oscillations. Transforming equations 2 using amplitude-phase complex variables similarly to what has been demonstrated in [6], the following first-order equation for the slowly varying amplitudes ($\ddot{A}_2 \approx 0$) can be produced:

$$2(\zeta_2 k_2 + j\omega)\dot{A}_2 + (k_2^2 - \omega^2 + 2j\omega k_2\zeta_2)A_2 = \\ = (j\omega g_2\Omega + 2j\omega\zeta_{12}k_1 + \dot{\Omega})A_1. \quad (3)$$

Here $A_1$ is the constant (does not depend on time) complex amplitude of the primary oscillations

$$A_1 = \frac{q_{10}}{k_1^2 - \omega^2 + 2jk_1\zeta_1\omega},$$

and complex amplitudes $A_i$ are expressed in terms of the real amplitudes and phases as $A_i(t) = A_{i0}(t)e^{j\varphi_{i0}(t)}$, where i equals 1 or 2 for the primary or secondary oscillations correspondingly. One should also note that disturbances in primary oscillations caused by secondary are considered negligible comparing to the forces from the excitation system.

Applying Laplace transformation to both sides of the equation 3, and solving obtained algebraic equation for the secondary amplitude, results in

$$A_2(s) = \frac{A_1\big[(s + jg_2\omega)\Omega(s) + 2\omega k_2\zeta_{12}(s)\big]}{k_2^2 - \omega^2 + 2j\omega k_2\zeta_2 + 2s(k_2\zeta_2 + j\omega)}. \quad (4)$$

Solution 4 can be represented as a sum of the following two components:

$$A_2(s) = A_2^\Omega(s) + A_2^\zeta(s),$$

$$A_2^\Omega(s) = \frac{A_1(j\omega g_2 + s)}{(s + j\omega)^2 + 2\zeta_2 k_2(s + j\omega) + k_2^2}\Omega(s), \quad (5)$$

$$A_2^\zeta(s) = \frac{2A_1 j\omega k_2}{(s + j\omega)^2 + 2\zeta_2 k_2(s + j\omega) + k_2^2}\zeta_{12}(s).$$

Here $A_2^\Omega(s)$ is part of the secondary amplitude due to the input angular rate, and $A_2^\zeta(s)$ is due to the cross-damping. Corresponding to (5) transfer functions are hence defined as

$$A_2(s) = W_2^\Omega(s) \cdot \Omega(s) + W_2^\zeta(s) \cdot \zeta_{12}(s),$$

$$W_2^\Omega(s) = \frac{A_1(j\omega g_2 + s)}{(s + j\omega)^2 + 2\zeta_2 k_2(s + j\omega) + k_2^2}, \quad (6)$$

$$W_2^\zeta(s) = \frac{2A_1 j\omega k_2}{(s + j\omega)^2 + 2\zeta_2 k_2(s + j\omega) + k_2^2}.$$

It is important to remember that part of the secondary amplitude due to the cross-damping will be undistinguishable from the one caused by the angular rate. Let us therefore derive transfer function relating input cross-damping to the output angular rate as

$$\Omega^\zeta(s) = W_\Omega^\zeta(s) \cdot \zeta_{12}(s), \quad (7)$$

where $\Omega^\zeta(s)$ is the measured erroneous angular rate caused by the cross-damping. Quite apparently unknown transfer function $W_\Omega^\zeta(s)$ can be expressed using transfer functions from (6) as

$$W_\Omega^\zeta(s) = \frac{W_2^\zeta(s)}{W_2^\Omega(s \to 0)} = \\ = \frac{2k_2(k_2^2 - \omega^2 + 2jk_2\omega\zeta_2)}{g_2(k_2^2 - \omega^2 + 2k_2\zeta_2 s + 2j\omega(s + k_2\zeta_2))}. \quad (8)$$

Transfer function (8) can be further simplified using assumptions that are relevant to CVGs with cylindrical sensitive element, and are good approximations for other sensitive elements designs (see [6]). Namely, we can assume that natural frequencies are equal ($k_1 = k_2 = k$) as well as relative damping coefficients ($\zeta_1 = \zeta_2 = \zeta$), and primary

oscillations excitation frequency is $\omega = k\sqrt{1-2\zeta^2}$. With these assumptions transfer function (8) becomes

$$W_\Omega^\zeta(s) = \frac{2k^2\zeta}{g_2(s+k\zeta)} .$$
(9)

Transfer function (9) allows efficient analysis of errors due to the cross-damping, which not only is present in the system, but can vary due to the different reasons.

## IV. EMPIRICAL MODELLING OF CROSS-DAMPING

Assuming that the cross-damping coefficient is a function of the temperature shift $T$ from the calibration temperature, it can be approximated using polynomial as

$$\zeta_{12} = \zeta_{12}(T) \approx \sum_{i=0}^{n} \zeta_i^T T^i .$$
(10)

Number of terms $n$ in representation (10) is selected to provide required accuracy.

Coefficients $\zeta_i^T$ can be determined experimentally when ambient temperature is known (measured) and angular rate is absent. Parameters of the cross-damping model (10) were found to have the following values: $\zeta_0^T = 1.0792\cdot10^{-3}$, $\zeta_1^T = -4.631\cdot10^{-5}$, $\zeta_2^T = 7.7044\cdot10^{-7}$, $\zeta_3^T = -5.8598\cdot10^{-9}$. Influence of the higher order components has been found negligible. In order to validate cross-damping model (10) we build temperature error compensation system using mentioned above parameters and model (9) of cross damping errors.

CVG output after temperature error compensation is shown in Fig. 2 (compensated).

## V. RESUME

Developed in this paper model of temperature related errors in CVGs along with the empirical model of the cross-damping have been used to develop temperature error compensation system, which significantly (approximately 8 times) improved CVG performances in terms of zero-rate output. In future research we plan to use model of cross-damping errors to develop stochastic system of temperature errors compensation.

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